Your UVA ID in capital letters:

Your full name:

Minimum-minimum background test

1. basic probability
   
a. True or False? If \( P(A \mid B) = P(A) \), then \( P(AB) = P(A)P(B) \)

b. If A and B are disjoint events, \( Pr(B) > 0 \), what is the value of \( Pr(A \mid B) \) ?

2. linear algebra

   Let \( x = (x_1, x_2, x_3)^T \) and:
   \[
   \begin{align*}
   2x_1 + 2x_2 + 3x_3 &= 1 \\
   x_1 - x_2 &= -1 \\
   -x_1 + 2x_2 + x_3 &= 2
   \end{align*}
   \]

   Write it into matrix form (i.e. \( Ax = b \))
Minimum background test

3. Discrete and Continuous Distributions

Match the distribution name to its formula.

- **Multivariate Gaussian**: \( p^x(1-p)^{1-x} \)
- **Exponential**: \( \frac{1}{b-a} \) when \( a \leq x \leq b; 0 \) otherwise
- **Uniform**: \( \frac{n}{x}p^x(1-p)^{n-x} \)
- **Bernoulli**: \( \lambda e^{-\lambda x} \) when \( x \geq 0; 0 \) otherwise
- **Binomial**: \( \frac{1}{\sqrt{2\pi d\Sigma}} \exp\left(-\frac{1}{2}(x - \mu)^\top\Sigma^{-1}(x - \mu)\right) \)

4. Probability and Random Variables

(a) State true or false. Here \( A^c \) denotes complement of the event \( A \).

- (a) \( P(A \cup B) = P(A \cap (B \cap A^c)) \)
- (b) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
- (c) \( P(A) = P(A \cap B) + P(A^c \cap B) \)
- (d) \( P(A|B) = P(B|A) \)
- (e) \( P(A_1 \cap A_2 \cap A_3) = P(A_3|(A_2 \cap A_1))P(A_2|A_1)P(A_1) \)

5. Prove one sentence explanation for the following terms:

- Derivative
- Gradient:
- Hessian Matrix
- Expectation of a Random Variable:
Although many students find the machine-learning class to be very rewarding, the class does assume that you have a basic familiarity with several types of math. Before taking the class, you should evaluate whether you have the mathematical background the class depends upon.

- Multivariate calculus (at the level of a first undergraduate course). For example, we rely on you being able to derive gradients of some multivariate functions.

- Linear algebra (at the level of a first undergraduate course). For example, we assume you know how to multiply vectors and matrices, and that you understand matrix inversion, eigenvectors and eigenvalues.

- Basic probability and statistics (at the level of a first undergraduate course). For example, we assume you know how to find the mean and variance of a set of data, you are familiar with common probability distributions such as the Gaussian and Uniform distributions, and that you understand basic notions such as conditional probabilities and Bayes rule. During the class, you might be asked to calculate the likelihood (probability) of a data set with respect to some given probability distribution, and to then derive the parameters of the distribution that maximize this data likelihood.