# UVA CS 4774: Machine Learning

# S5: Lecture 25 Extra: Unsupervised Clustering (III): Gaussian Mixture Model

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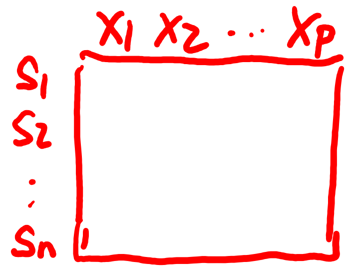
# Course Content Plan → Regarding Tasks

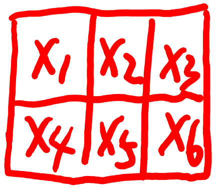
- ~~Regression (supervised)~~ ← Y is a continuous
- ~~Learning theory~~ ← About f()
- ~~Classification (supervised)~~ ← Y is a discrete
- Unsupervised models ← NO Y
- ~~Graphical models~~ ← About interactions among Y,X1,. Xp
- Reinforcement Learning ← Learn to Interact with environment

# Course Content Plan ➔ Regarding Data

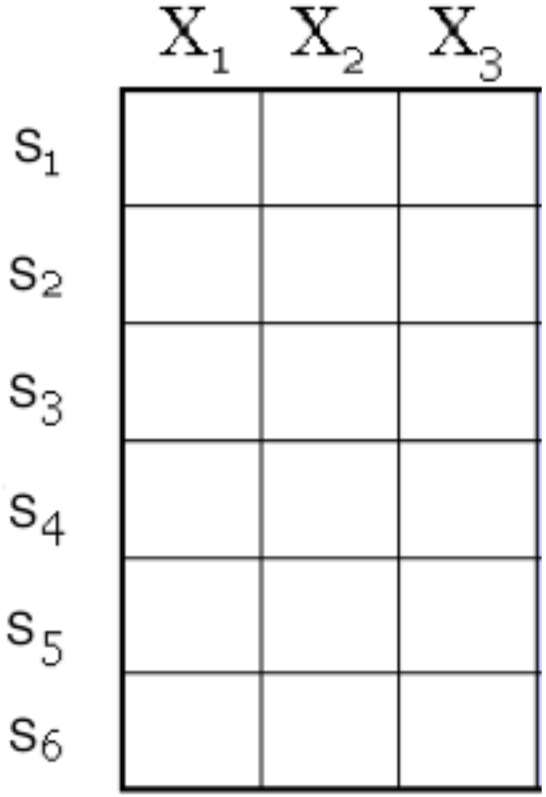
- ❑ Tabular / Matrix




- ❑ 2D Grid Structured: Imaging




- ❑ 1D Sequential Structured: Text
- ❑ Graph Structured (Relational)
- ❑ Set Structured / 3D /




# An unlabeled Dataset X

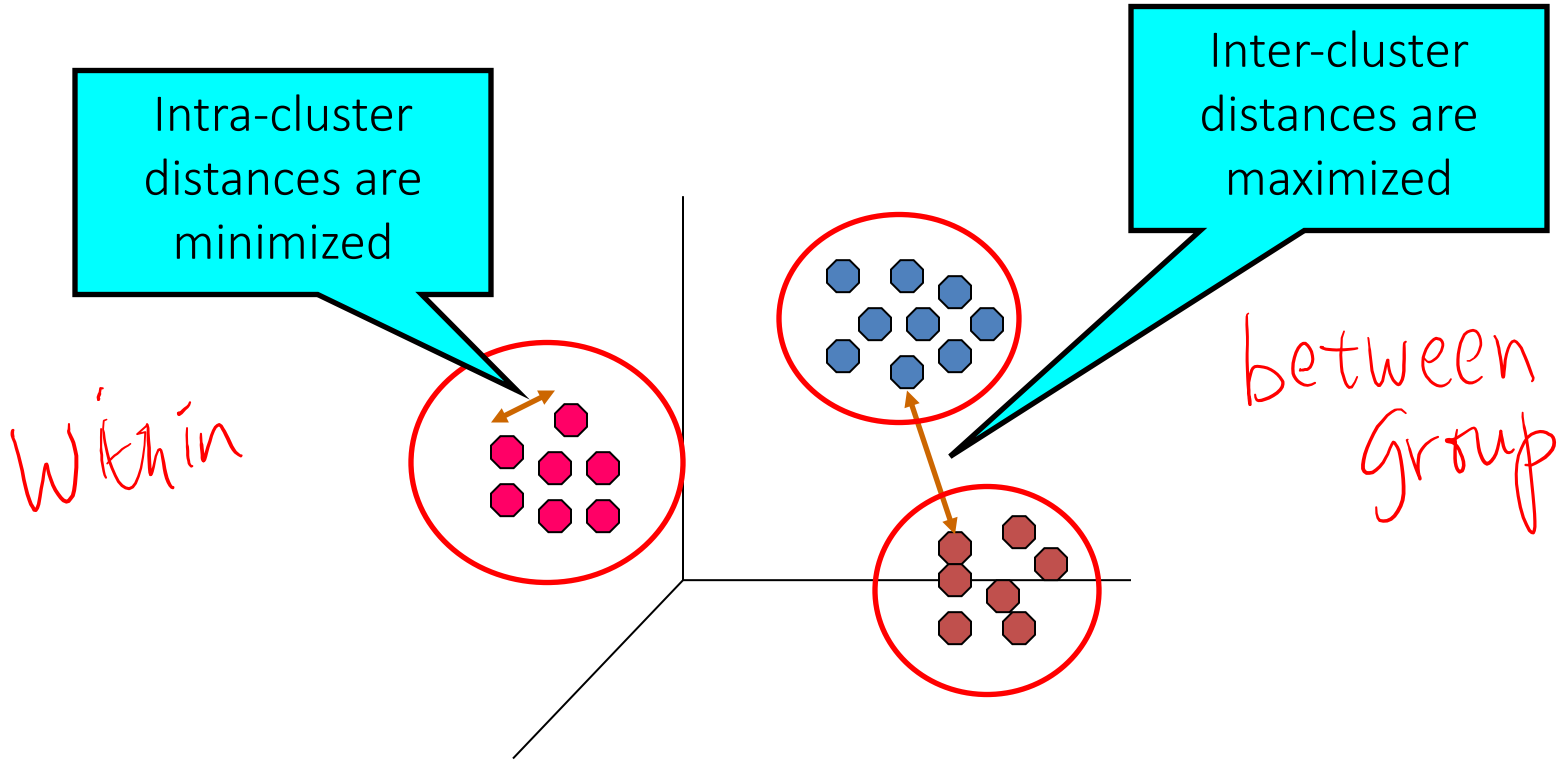
a data matrix of $n$ observations on $p$ variables $x_1, x_2, \ldots x_p$

Unsupervised learning = learning from raw (unlabeled, unannotated, etc) data, as opposed to supervised data where a classification label of examples is given

- **Data**/*points/instances/examples/samples/records*: [ rows ]
- **Features**/*attributes/dimensions/independent variables/covariates/predictors/regressors*: [ columns]

# What is clustering?

- Find groups (clusters) of data points such that data points in a group will be similar (or related) to one another and different from (or unrelated to) the data points in other groups

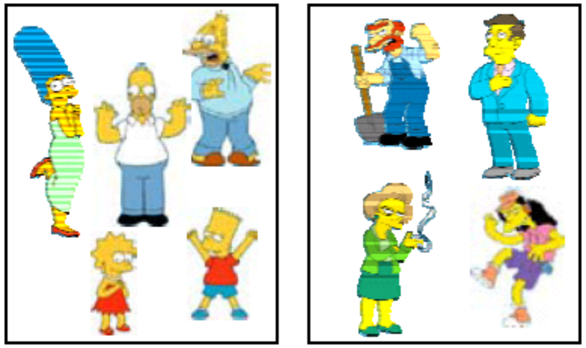


# Roadmap: clustering

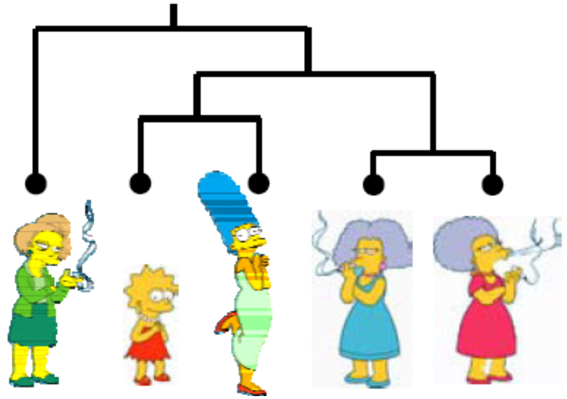
- Definition of "groupness”
- Definition of "similarity/distance"
- Representation for objects
- How many clusters?
- Clustering Algorithms
  - Partitional algorithms
  - Hierarchical algorithms
- Formal foundation and convergence

# Clustering Algorithms

- Partitional algorithms
  - Usually start with a random (partial) partitioning
  - Refine it iteratively
    - K means clustering
    - Mixture-Model based clustering

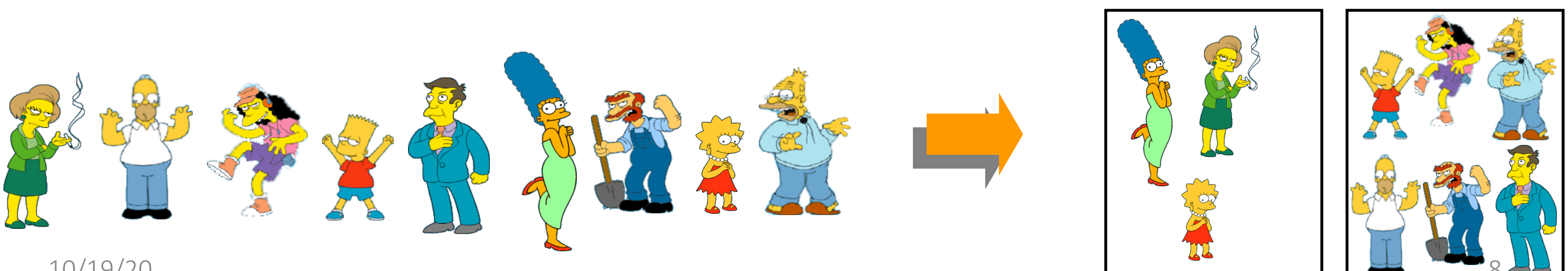


- Hierarchical algorithms
  - Bottom-up, agglomerative
  - Top-down, divisive



# (2) Partitional Clustering

- Nonhierarchical
- Construct a partition of n objects into a set of K clusters
- User has to specify the desired number of clusters K.



# Other partitioning Methods

- Partitioning around medoids (PAM): instead of averages, use multidim medians as centroids (cluster “prototypes”). Dudoit and Freedland (2002).

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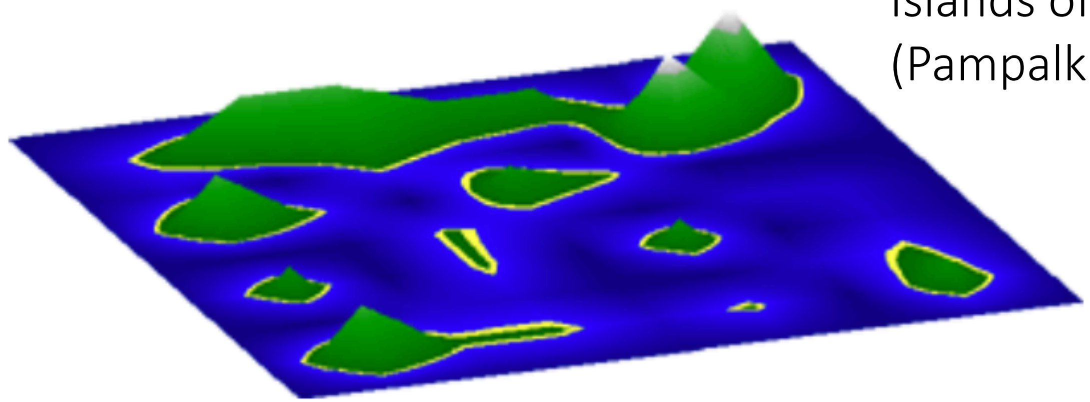
# E.g.: SOM Used for Visualization

## Islands of Music

### Analysis, Organization, and Visualization of Music Archives

Islands of music
(Pampalk et al., KDD' 03)



**piece of music**: member of a *music collection* and inhabitant of *islands of music*. Groups of similar pieces of music (also known as *genres*) like to gather around large mountains or small hills depending on the size of the group. Groups which are similar to each other like to live close together. Individuals which are not members of specific groups usually live near the beach and some very individualistic pieces might be found swimming in deep water.

**islands of music**: serve as graphical *user interface* to a music collection and are intended to help the user explore vast amounts of music in an efficient way. Islands of music are generated automatically based on *psychoacoustics models* and *self-organizing maps*.

# Other partitioning Methods

- Partitioning around medoids (PAM): instead of averages, use multidim medians as centroids (cluster “prototypes”). Dudoit and Freedland (2002).
- Self-organizing maps (SOM): add an underlying “topology” (neighboring structure on a lattice) that relates cluster centroids to one another. Kohonen (1997), Tamayo et al. (1999).
- Fuzzy k-means: allow for a “gradation” of points between clusters; soft partitions. Gash and Eisen (2002).

# Other partitioning Methods

$C_j \in$ train Set

- Partitioning around medoids (PAM): instead of averages, use multidim medians as centroids (cluster "prototypes"). Dudoit and Freedland (2002).
- Self-organizing maps (SOM): add an underlying "topology" (neighboring structure on a lattice) that relates cluster centroids to one another. Kohonen (1997), Tamayo et al. (1999).
- Fuzzy k-means: allow for a "gradation" of points between clusters; soft partitions. Gash and Eisen (2002).
- Mixture-based clustering: implemented through an EM (Expectation-Maximization)algorithm. This provides soft partitioning, and allows for modeling of cluster centroids and shapes. (Yeung et al. (2001), McLachlan et al. (2002))

$m_{ij} \in \{1, 0\} \rightarrow [0, 1]$

# (3) GMM Clustering



Data → Task → Representation → Score Function → Search/Optimization → Models, Parameters

Tabular → Clustering → Mixture of Gaussian → Likelihood → EM algorithm → Each point's soft membership & mean / covariance per cluster

$$\sum_i \log \prod_{i=1}^{n} p(x=x_i) = \sum_i \log \left[ \sum_{\mu_j} p(\mu=\mu_j) \frac{1}{(2\pi)\left|\Sigma_j\right|^{1/2}} e^{-\frac{1}{2}\left(\vec{x}-\vec{\mu}_j\right)^T \Sigma_j^{-1}\left(\vec{x}-\vec{\mu}_j\right)} \right]$$

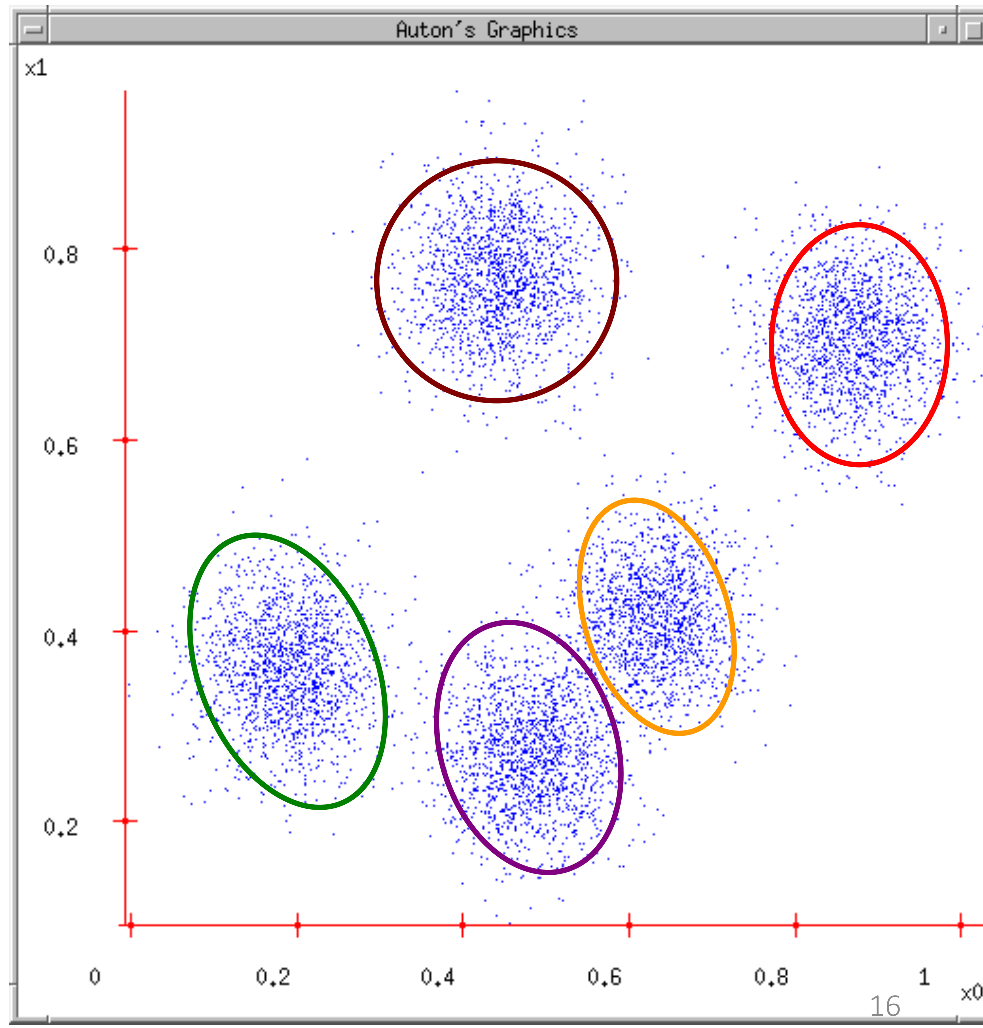
# Partitional : Gaussian Mixture Model

- 1. Review of Gaussian Distribution
- 2. GMM for clustering : basic algorithm
- 3. GMM connecting to K-means
- 4. Problems of GMM and K-means

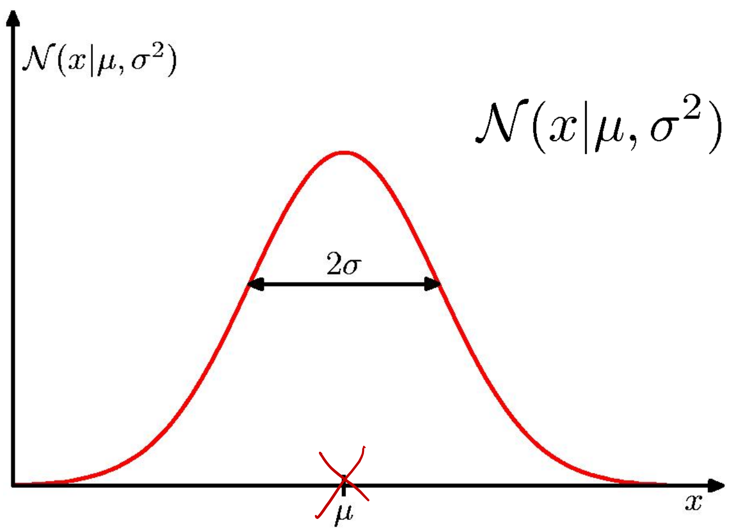
# A Gaussian Mixture Model for Clustering

- Assume that data are generated from a mixture of Gaussian distributions
- For each Gaussian distribution
  - Center: $\mu_j$
  - covariance: $\Sigma_j$
- For each data point
  - Determine membership

$$z_{ij} : \text{ if } x_i \text{ belongs to j-th cluster}$$

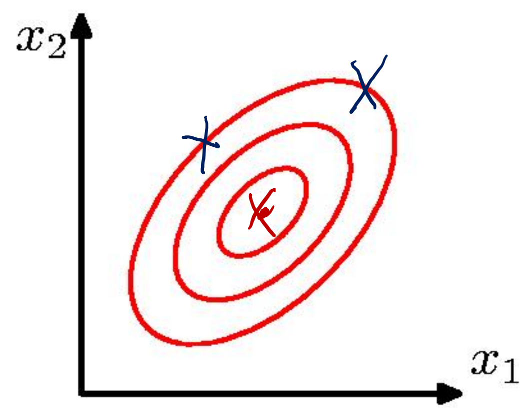


# Gaussian Distribution




$$\mathcal{N}(x|\mu,\sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$$

$$X \sim N\left(\mu, \sigma^2\right)$$




$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

Mean

Covariance Matrix



Courtesy: http://research.microsoft.com/~cmbishop/PRML/index.htm

# Example: the Bivariate Normal distribution

$$p(\vec{x}) = f\left(x_1, x_2\right) = \frac{1}{\left(2\pi\right)\left|\Sigma\right|^{1/2}} e^{-\frac{1}{2}\left(\vec{x}-\vec{\mu}\right)^T \Sigma^{-1}\left(\vec{x}-\vec{\mu}\right)}$$

with $\vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$ 2x1 and

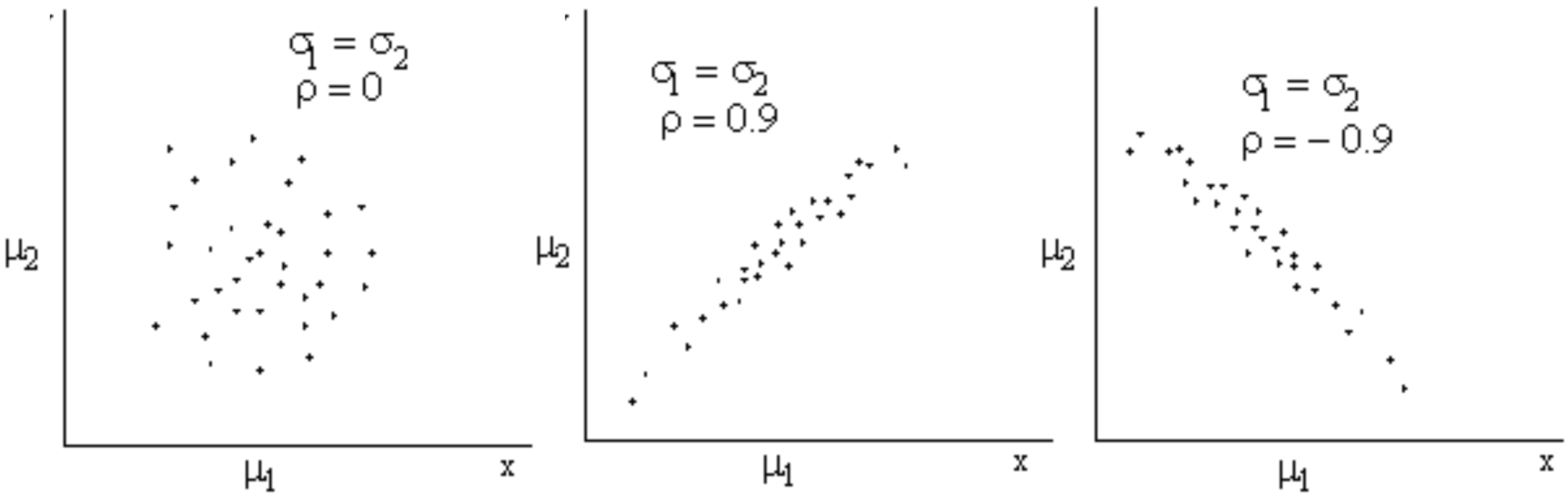
V(X1) Cov(X1, X2)

$$\underset{2\times 2}{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \underline{\rho\sigma_1\sigma_2} \\ \underline{\rho\sigma_1\sigma_2} & \sigma_2^2 \end{bmatrix}$$

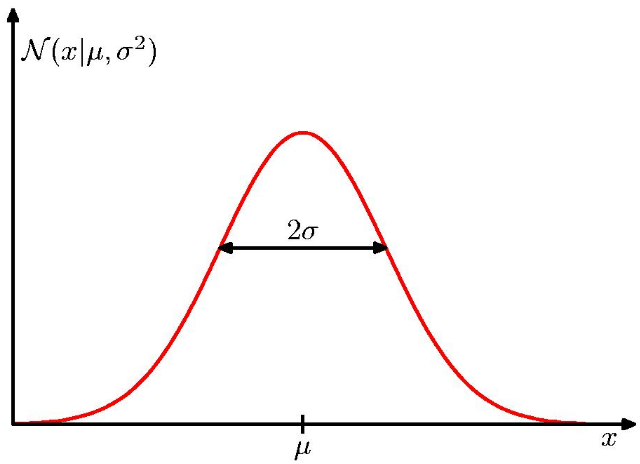
V(X2) . 2 X 2

$$\left|\Sigma\right| = \sigma_{11}\sigma_{22} - \sigma_{12}^2 = \sigma_1^2\sigma_2^2\left(1-\rho^2\right)$$

# Scatter Plots of data from the bivariate Normal distribution



# How to Estimate Gaussian: MLE




- In the 1D Gaussian case, we simply set the mean and the variance to the sample mean and the sample variance:

$$\overline{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\overline{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{\mu}\right)^2$$

# The p-multivariate Normal distribution

$$< X_1, X_2 \cdots, X_p > \sim N\left(\vec{\mu}, \Sigma\right)$$

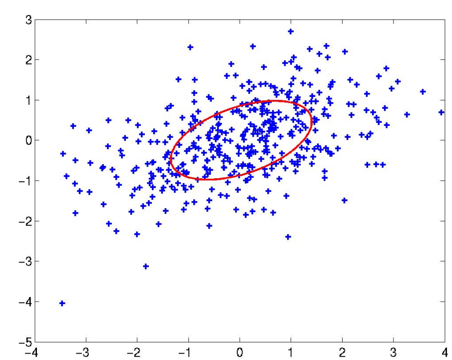
$$\vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix}_{p \times 1}$$

$$\mu_i = \frac{1}{n} \sum_{j=1}^{N} X_j^{(i)}$$

i-th feature $\in \{1, 2, \cdots, p\}$

j-th sample $\in \{1, 2, \cdots, N\}$

$$\Sigma_{P \times P} = \begin{array}{cc} & \begin{array}{ccc} \phantom{Var(X_1)} & & i \end{array} \\ \begin{array}{c} \\ j \\ \\ \end{array} & \begin{bmatrix} Var(X_1) & & Cov(X_i, X_j) \\ & \ddots & \\ & & Var(X_p) \end{bmatrix} \end{array}$$



# Partitional : Gaussian Mixture Model

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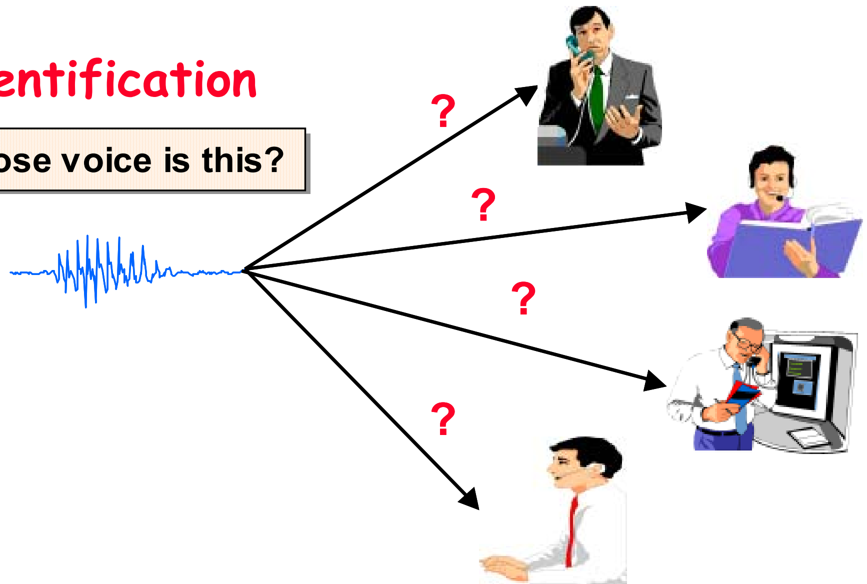
# Application: Three Speaker Recognition Tasks

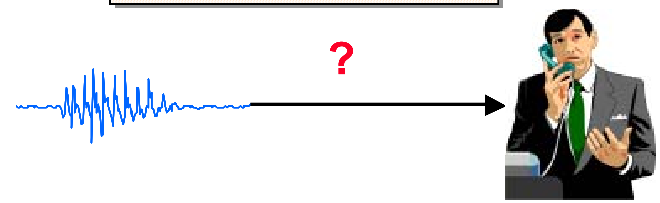
## Identification

Whose voice is this?




## Verification/Authentication/ Detection

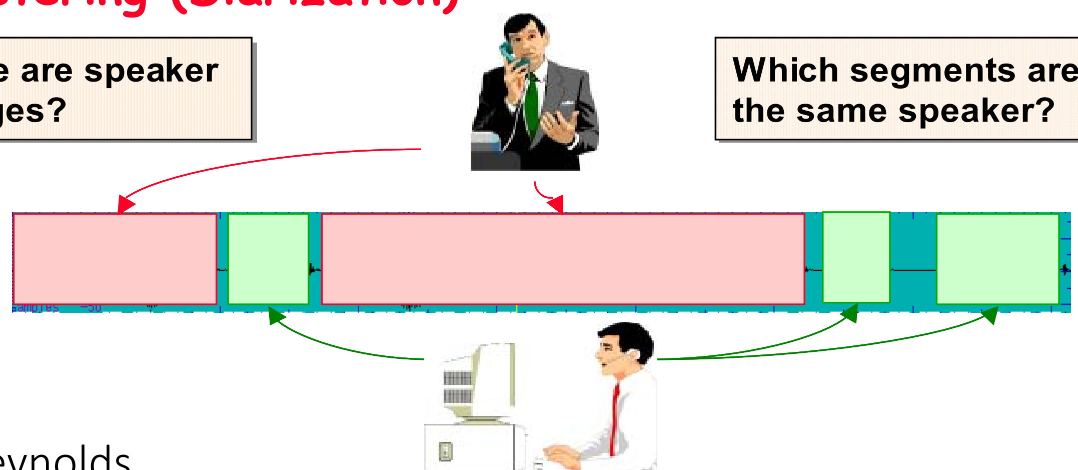
Is this Bob's voice?




## Segmentation and Clustering (Diarization)

Where are speaker changes?

Which segments are from the same speaker?





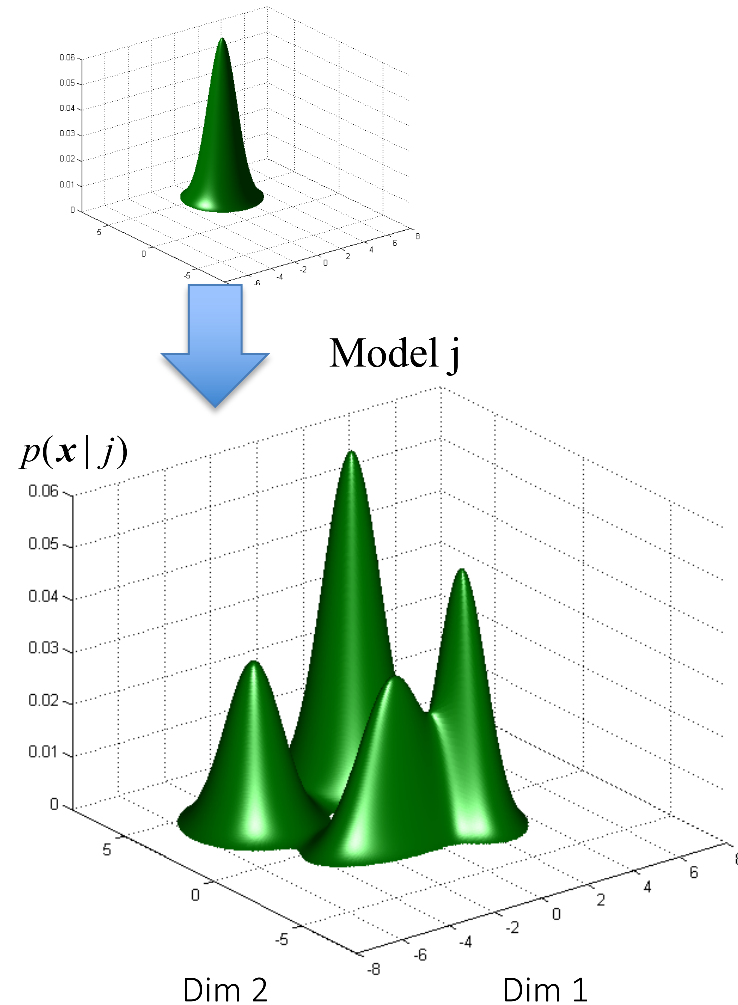
slide from Douglas Reynolds

# Application : GMMs for speaker recognition

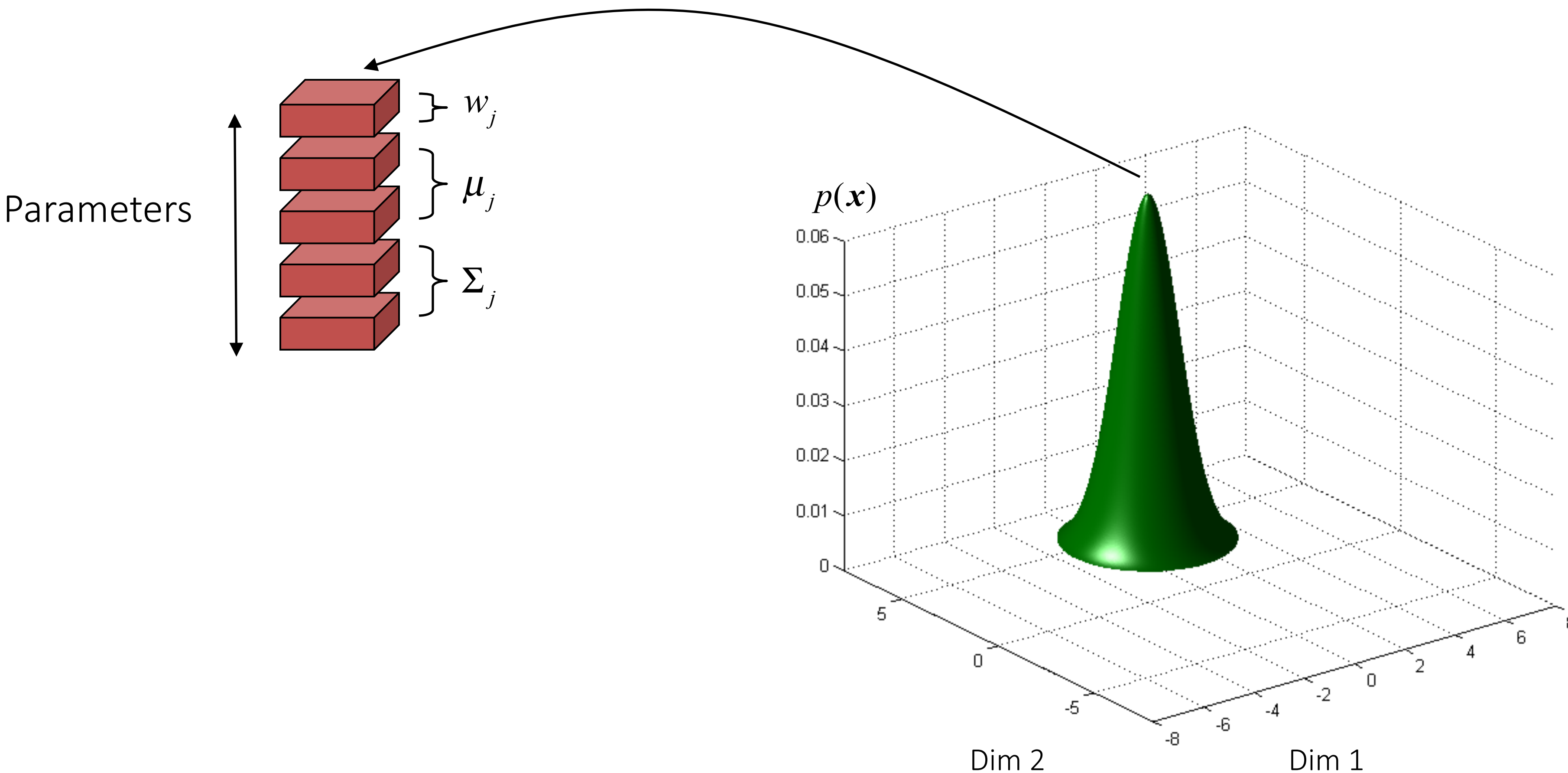
- A Gaussian mixture model (GMM) represents as the weighted sum of multiple Gaussian distributions

- Each Gaussian state i has a
  - Mean $\mu_j$
  - Covariance $\Sigma_j$
  - Weight

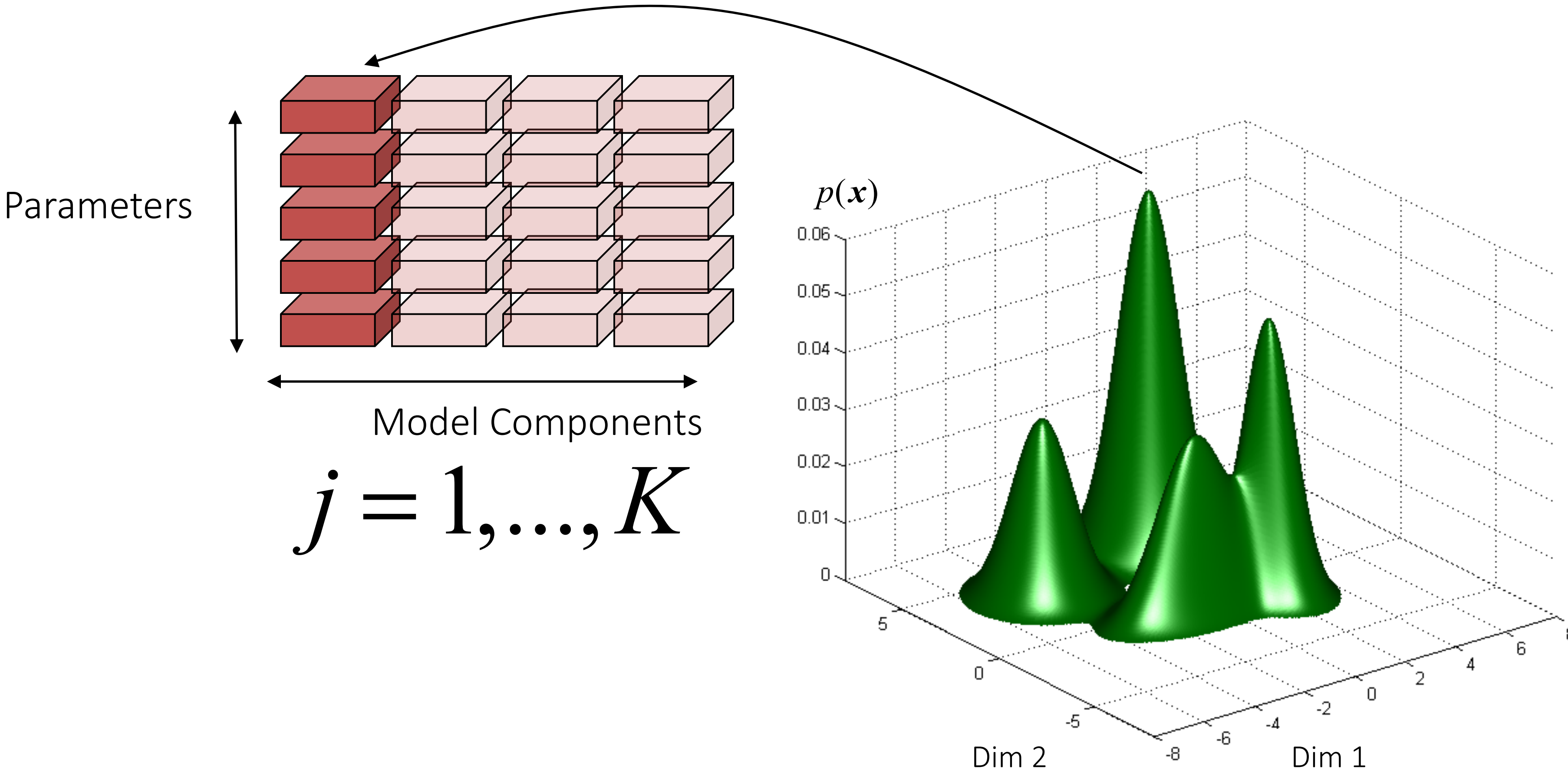
$$w_j \equiv p(\mu = \mu_j)$$



# Recognition Systems
# Gaussian Mixture Models



# Recognition Systems
# Gaussian Mixture Models




$$j = 1, \ldots, K$$

# Learning a Gaussian Mixture

- Probability Model

A Gaussian mixture model (GMM) represents as the weighted sum of multiple Gaussian distributions

$$p(\vec{x}=\vec{x}_i)$$

$$=\sum_j p(\vec{x}=\vec{x}_i,\vec{\mu}=\vec{\mu}_j)$$

Total low of probability

$$=\sum_j p(\vec{\mu}=\vec{\mu}_j)p(\vec{x}=\vec{x}_i\,|\,\vec{\mu}=\vec{\mu}_j)$$

Chain rule

$$=\sum_j p(\vec{\mu}=\vec{\mu}_j)\frac{1}{(2\pi)^{p/2}\left|\Sigma_j\right|^{1/2}}\mathrm{e}^{-\frac{1}{2}\left(\vec{x}-\vec{\mu}_j\right)^T\Sigma_j^{-1}\left(\vec{x}-\vec{\mu}_j\right)}$$

# Max Log-likelihood of Observed Data Samples

- Log-likelihood of data $logp(x_1, x_2, x_3, ..., x_n) =$

$$\log \prod_{i=1..n} \sum_{j=1..K} p(\vec{\mu} = \vec{\mu}_j) \frac{1}{(2\pi)^{p/2} |\Sigma_j|^{1/2}} e^{-\frac{1}{2}(\vec{x}_i - \vec{\mu}_j)^T \Sigma_j^{-1} (\vec{x}_i - \vec{\mu}_j)}$$



Apply MLE to find $\left\{ \{p(\vec{\mu} = \mu_j)\}, j = 1...K \right\}$

optimal Gaussian parameters $\{\vec{\mu}_j, \Sigma_j, j = 1...K\}$

# Expectation-Maximization for training GMM

- Start:
  - "Guess" the centroid and covariance for each of the K clusters
  - “Guess” the proportion of clusters, e.g., uniform prob 1/K

- Loop
  - For each point, revising its proportions belonging to each of the K clusters
  - For each cluster, revising both the mean (centroid position) and covariance (shape)

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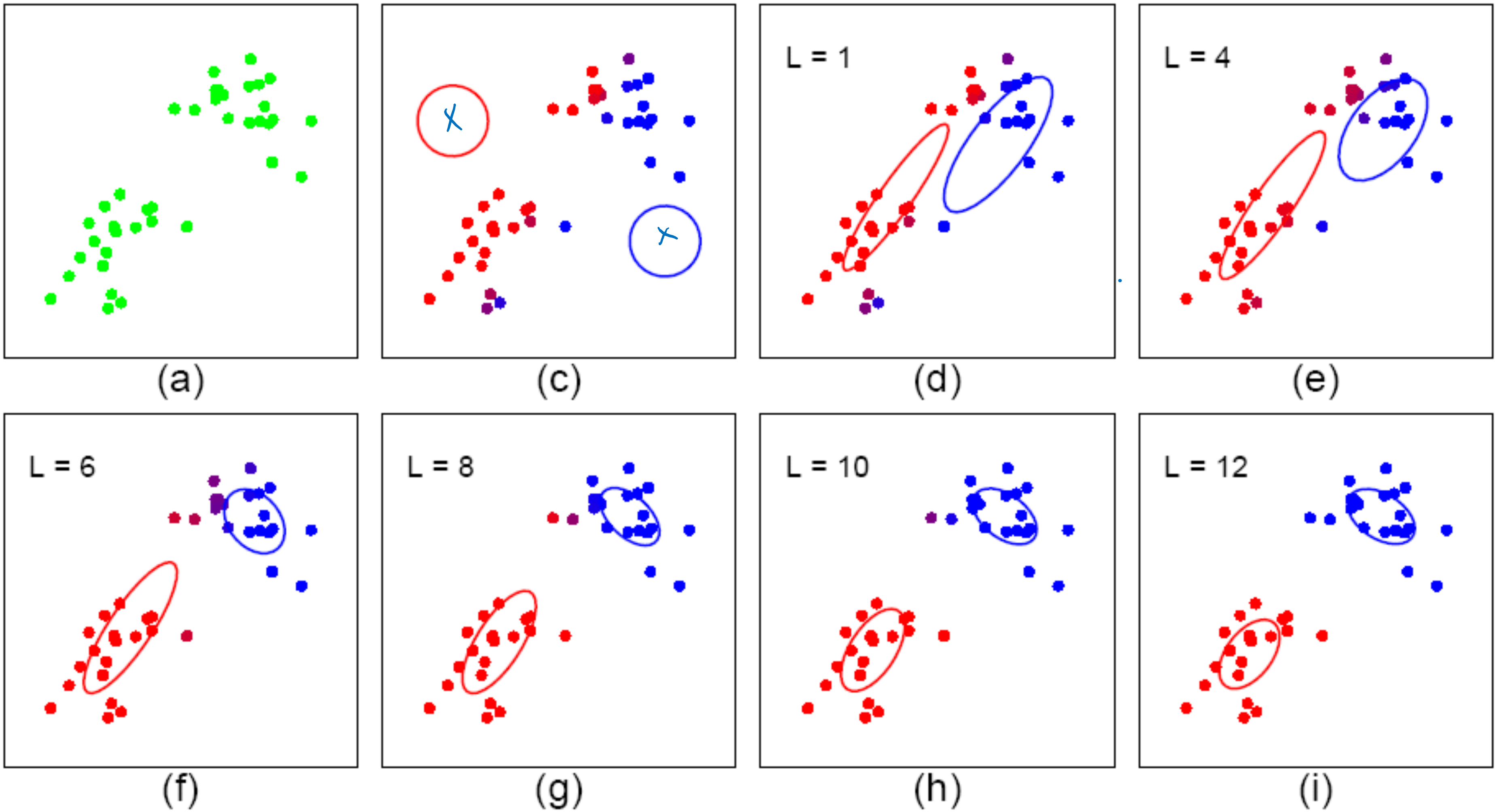
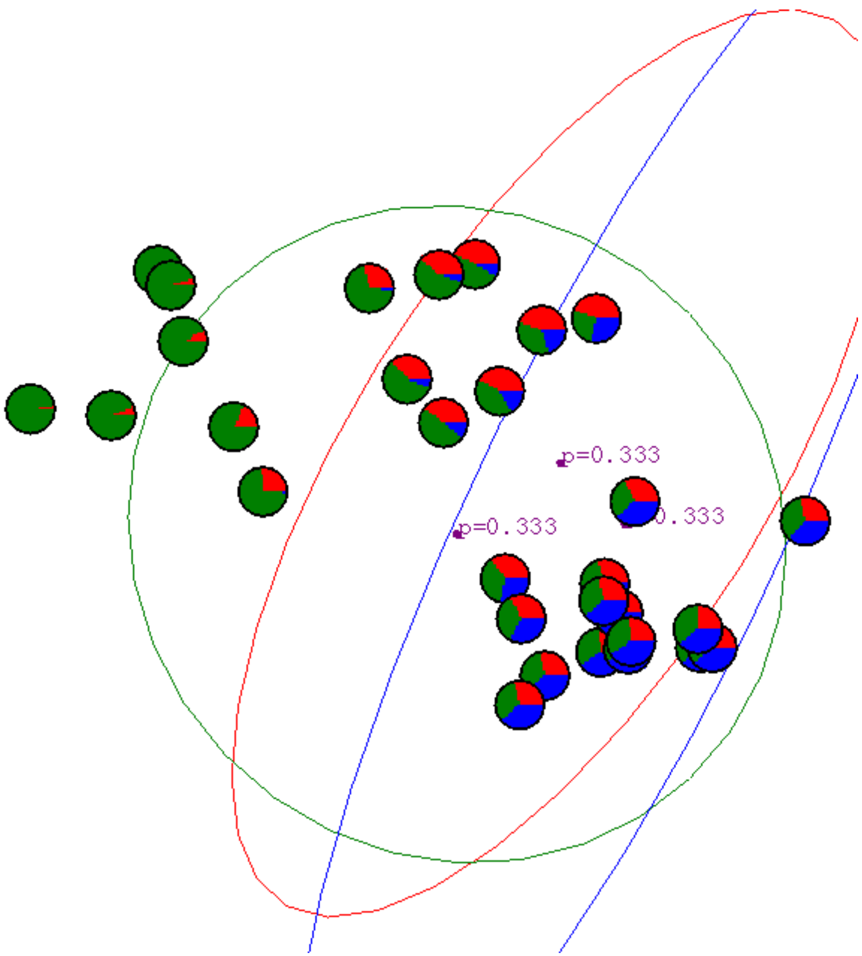


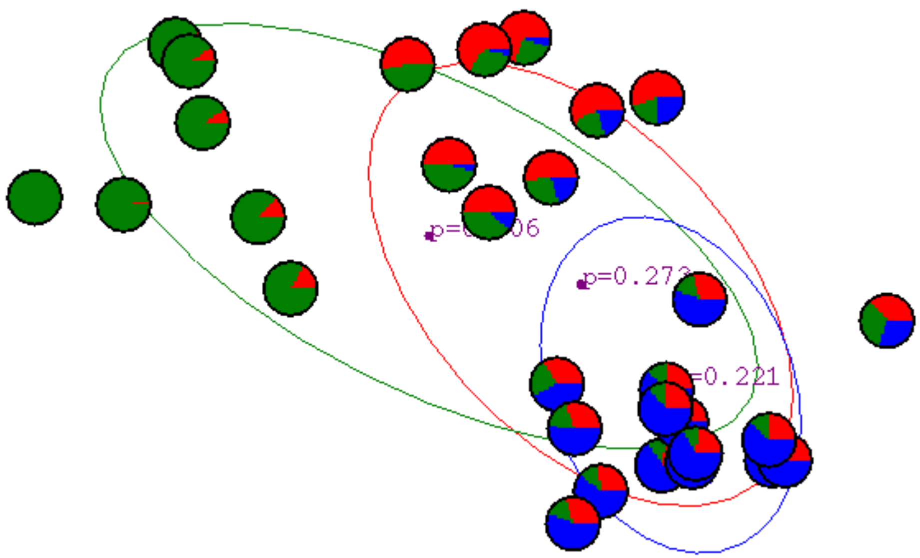
# Another Gaussian Mixture Example: Start




$p(\mu_j | x(i))$

# Another GMM Example: After First Iteration

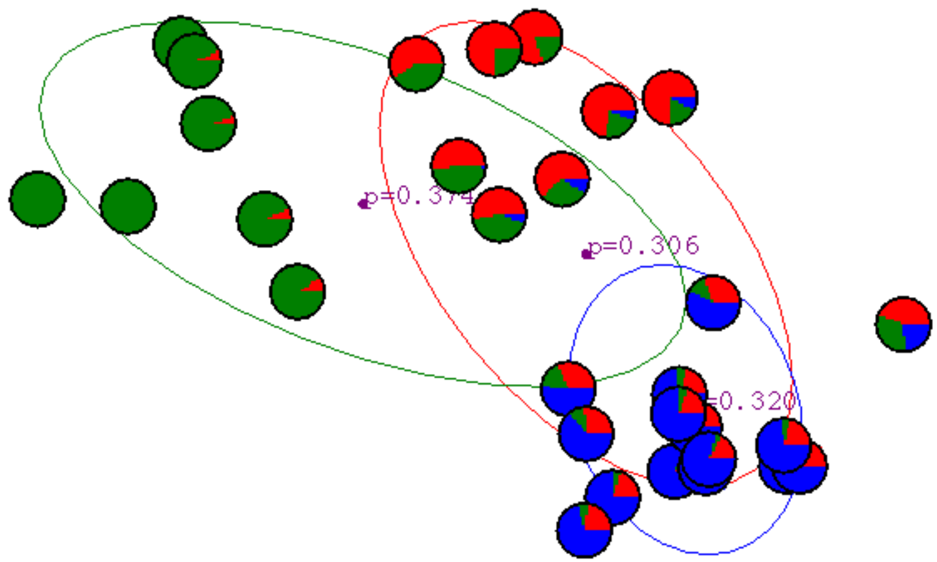
For each point, revising its proportions belonging to each of the K clusters




For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# Another GMM Example: After 2nd Iteration

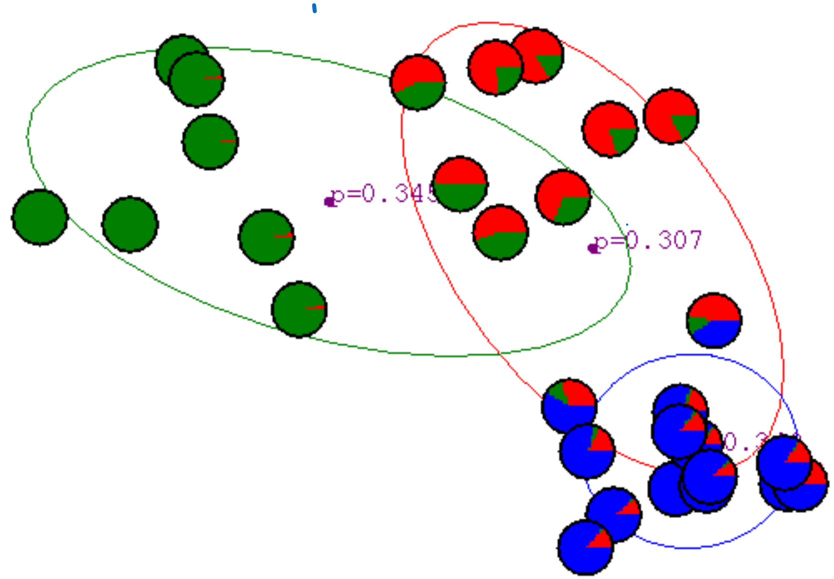
For each point, revising its proportions belonging to each of the K clusters




For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# After 3rd Iteration

For each point, revising its proportions belonging to each of the K clusters

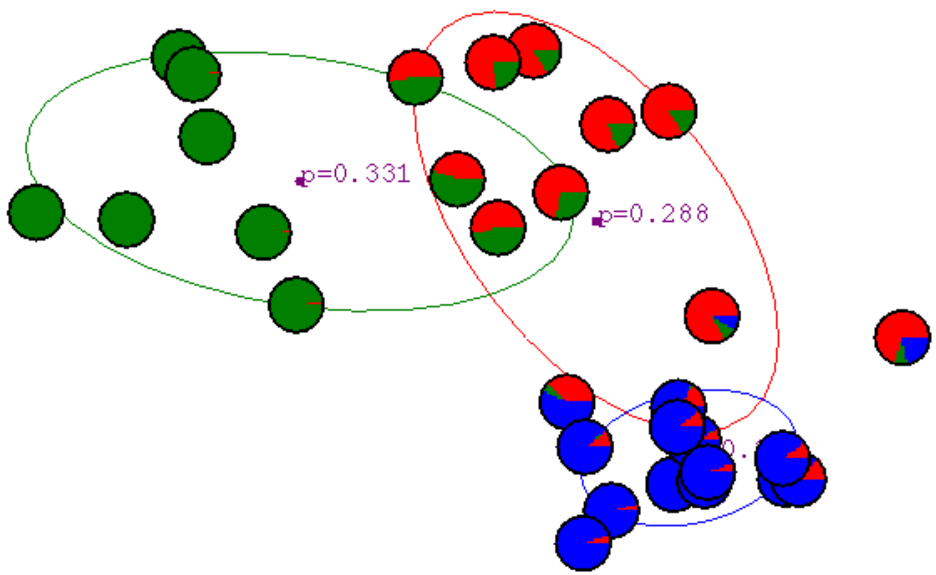



$p(\mu_j)$

For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# After 4th Iteration

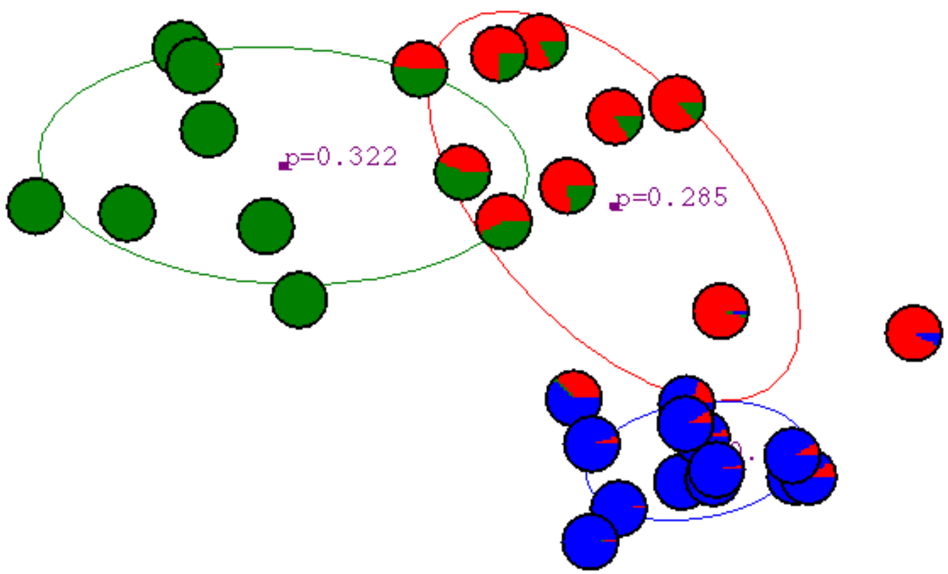
For each point, revising its proportions belonging to each of the K clusters




For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# After 5th Iteration

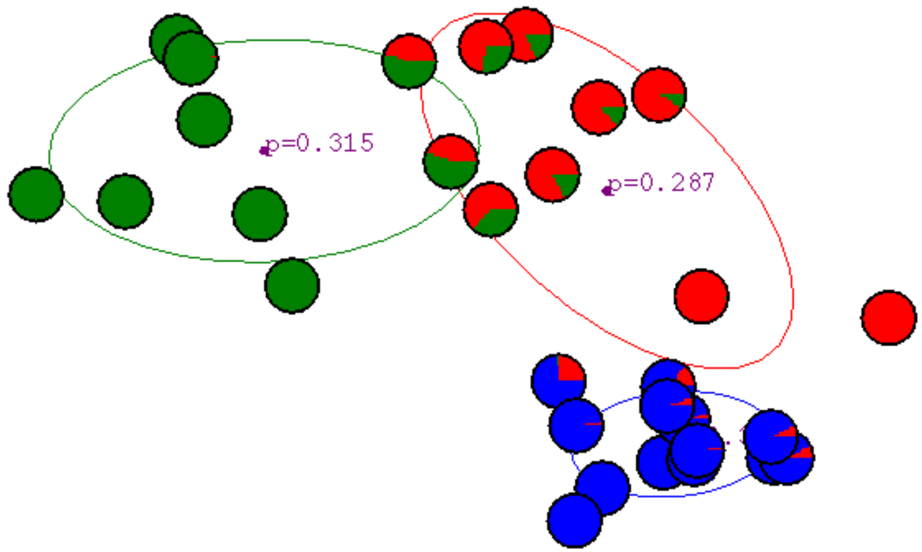
For each point, revising its proportions belonging to each of the K clusters




For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# After 6th Iteration

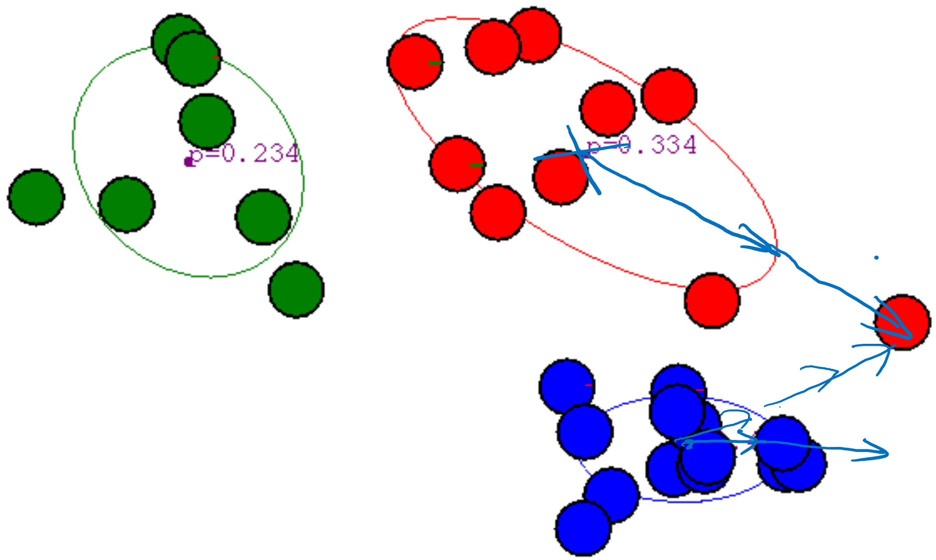
For each point, revising its proportions belonging to each of the K clusters




For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

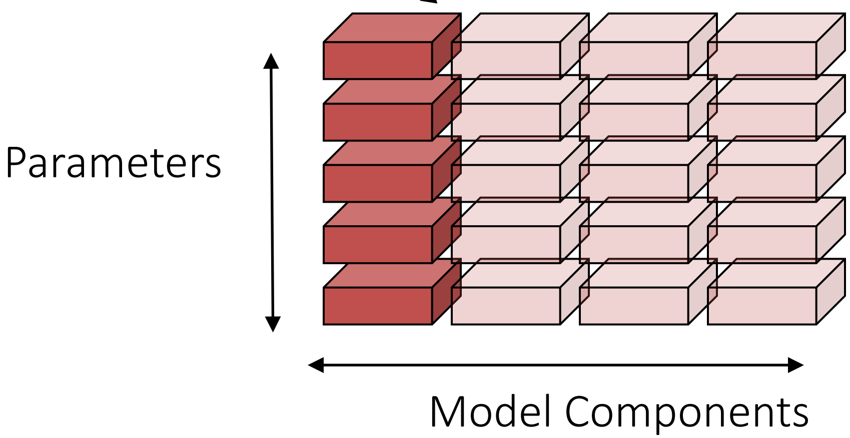
# Another GMM Example: After 20th Iteration

For each point, revising its proportions belonging to each of the K clusters

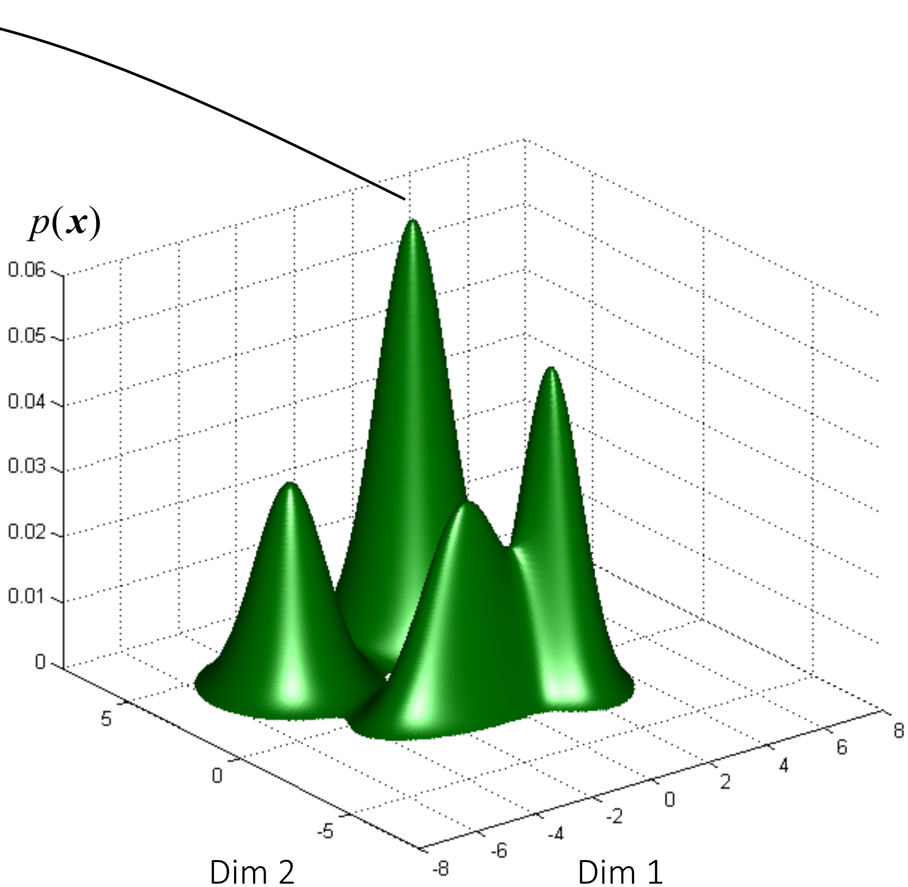



For each cluster, revising its mean (centroid position), covariance (shape) and proportion in the mixture

# Recap: Gaussian Mixture Models

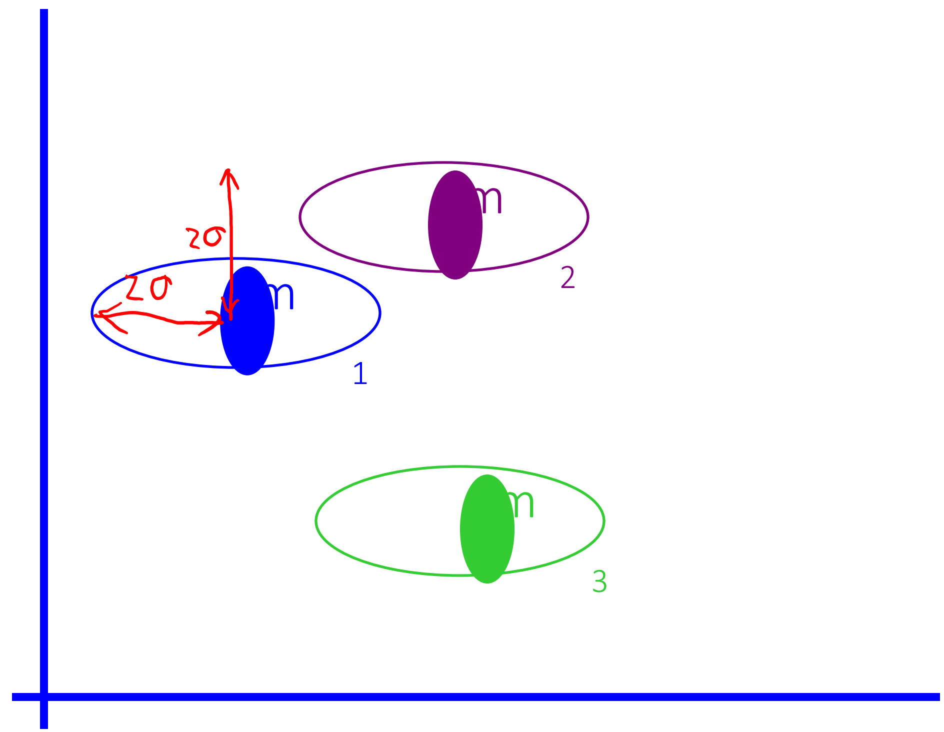



$$j = 1, \ldots, K$$



# The Simplest GMM assumption

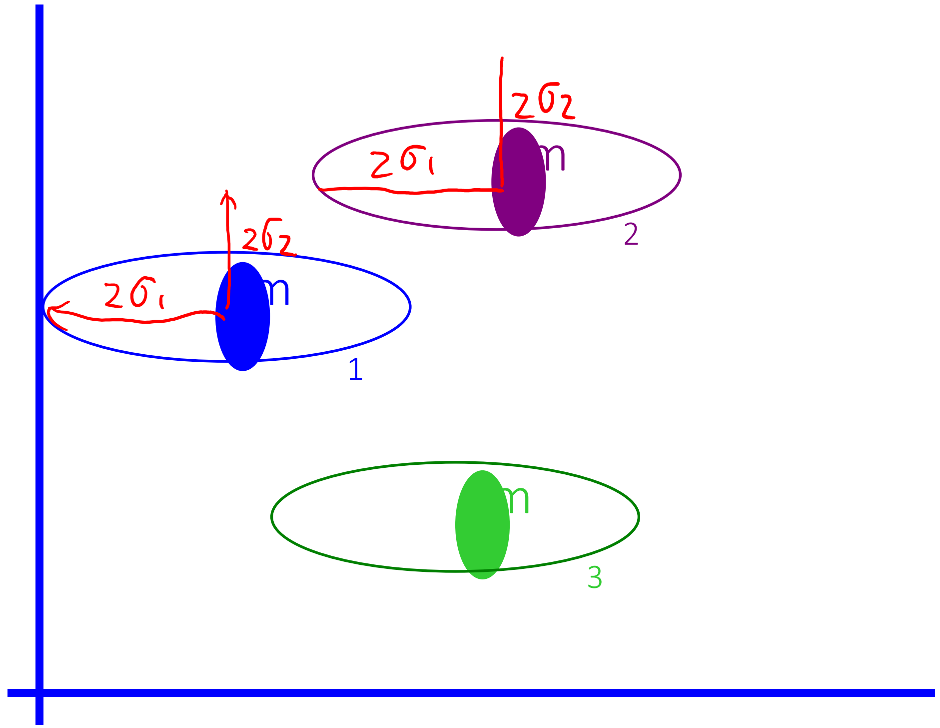
- Each component generates data from a Gaussian with
  - mean $\mu_i$
  - Shared diagonal covariance matrix $\sigma^2 I$




$$\Sigma_j = \Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$$

# Another Simple GMM assumption

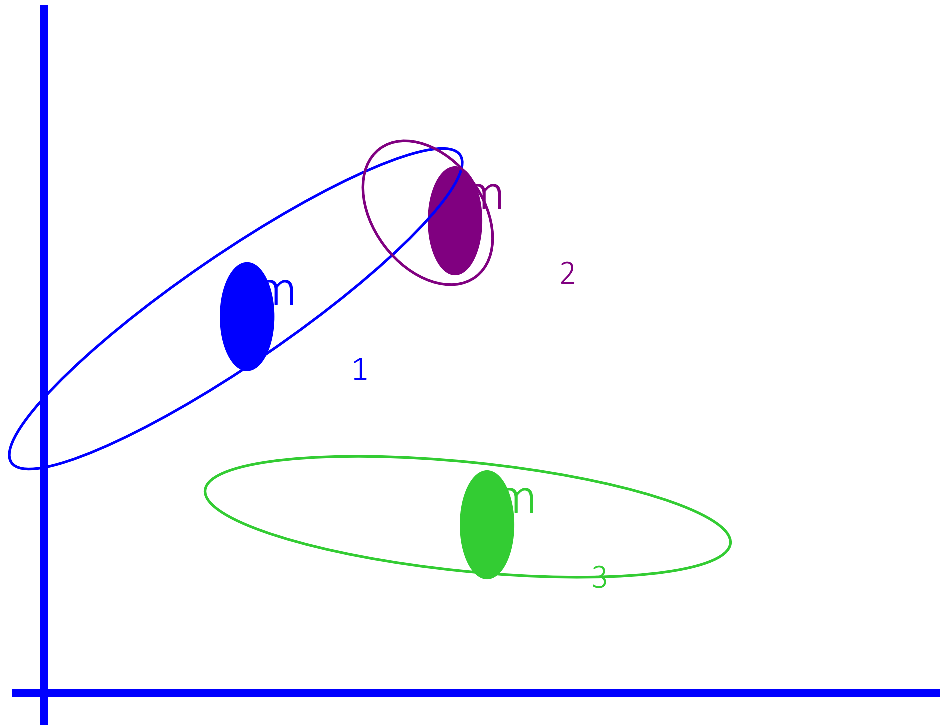
- Each component generates data from a Gaussian with
  - mean $\mu_i$
  - Shared covariance matrix as diagonal matrix




$$\Sigma_j = \Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

# The General GMM assumption

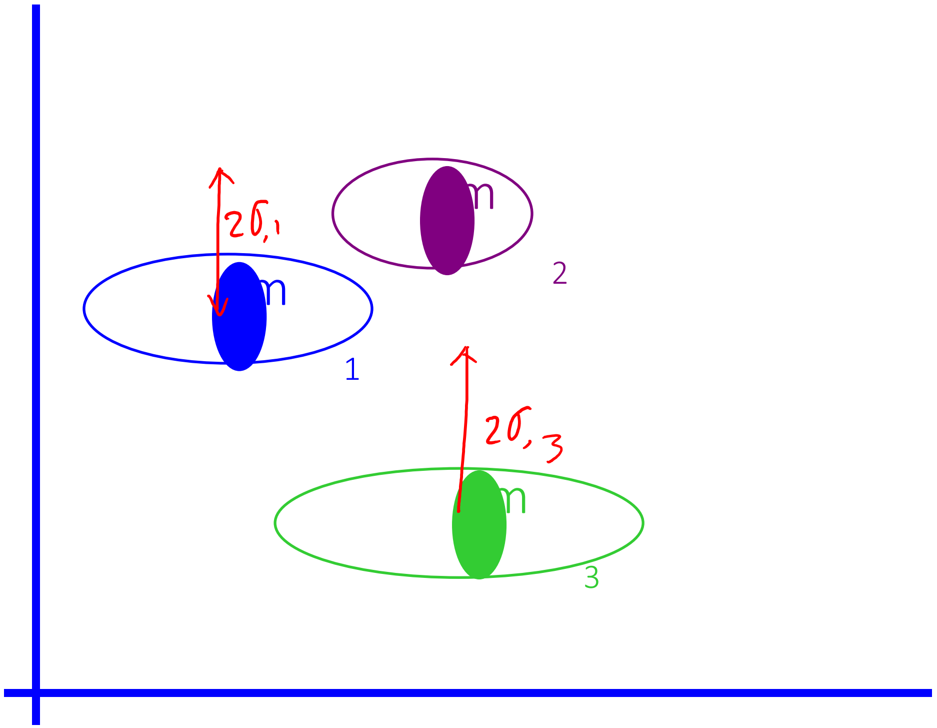
- Each component generates data from a Gaussian with
  - mean $\mu_i$
  - covariance matrix $\Sigma_i$




$$\Sigma_j = \begin{bmatrix} \sigma_{1j} & cov_j(X_1, X_2) \\ cov_j(X_1, X_2) & \sigma_{2j} \end{bmatrix}$$

# Another Simple GMM assumption

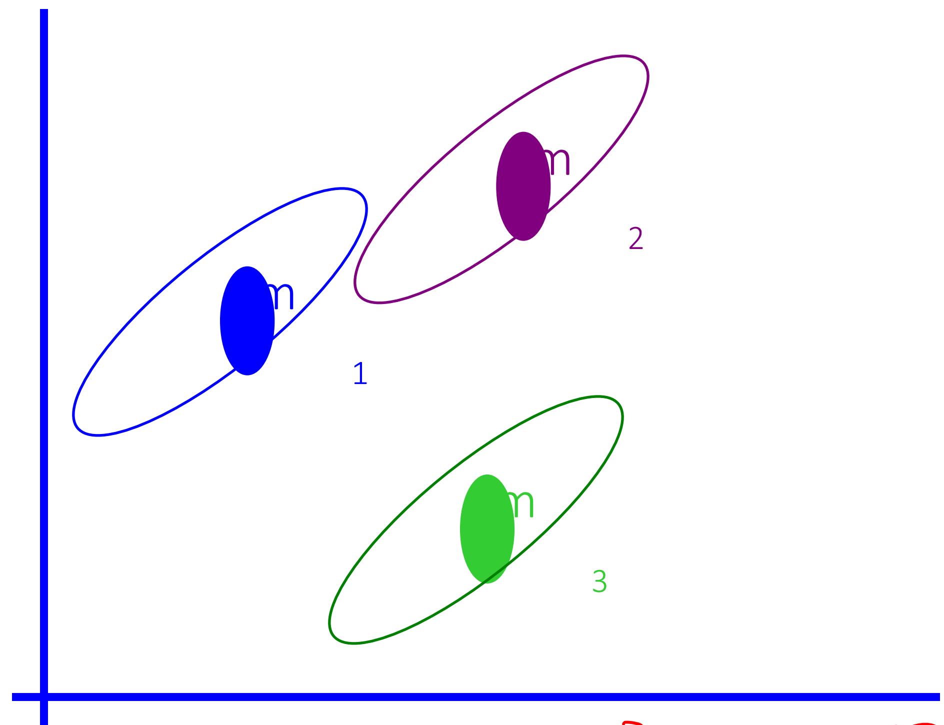
- Each component generates data from a Gaussian with
  - mean $\mu_i$
  - Cluster-specific diagonal covariance matrix as $\sigma_j^2 I$




$$\Sigma_j = \sigma_j^2 I = \begin{bmatrix} \sigma_j^2 & 0 \\ 0 & \sigma_j^2 \end{bmatrix}$$

# A bit More General GMM assumption

- Each component generates data from a Gaussian with
  - mean $\mu_i$
  - Shared covariance matrix as full matrix




$$\Sigma_j = \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

# Concrete Equations for Learning a Gaussian Mixture

(when assuming with known shared covariance

$$
\begin{aligned}
p(\vec{x} = \vec{x}_i) &= \sum_{\mu_j} p(\vec{x} = \vec{x}_i, \vec{\mu} = \vec{\mu}_j) \\
&= \sum_{j} p(\vec{\mu} = \vec{\mu}_j) p(\vec{x} = \vec{x}_i \mid \vec{\mu} = \vec{\mu}_j) \\
&= \sum_{j} p(\vec{\mu} = \vec{\mu}_j) \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}\left(\vec{x}_i - \vec{\mu}_j\right)^T \Sigma^{-1} \left(\vec{x}_i - \vec{\mu}_j\right)}
\end{aligned}
$$

Assuming Known and Shared

# Learning a Gaussian Mixture

(when assuming with known shared covariance)

[Bayes Rule]

E-Step

$$E[z_{ij}] = p(\vec{\mu} = \mu_j \mid x = x_i)$$

assignment. soft

$$= \frac{p(x = x_i \mid \mu = \mu_j)\, p(\mu = \mu_j)}{\sum_{s=1}^{k} p(x = x_i \mid \mu = \mu_s)\, p(\mu = \mu_s)}$$

$$= \frac{\frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(\vec{x}_i - \vec{\mu}_j)^T \Sigma^{-1} (\vec{x}_i - \vec{\mu}_j)} \quad p(\mu = \mu_j)}{\sum_{s=1}^{k} \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(\vec{x}_i - \vec{\mu}_s)^T \Sigma^{-1} (\vec{x}_i - \vec{\mu}_s)} \quad p(\mu = \mu_s)}$$

# E-step (vs. Assignment Step in K-means)

when assuming with known shared covariance

$m_{ij} = \begin{cases} 0 \\ 1 \end{cases}$

E-Step

$$E[z_{ij}] = p(\mu = \mu_j \mid x = x_i)$$

Soft assignment
$p(\mu = \mu_j \mid x = x_i)$
How $x_i$ belongs
in proportion
to cluster $\{1, 2, \ldots, k\}$

$$= \frac{p(x = x_i \mid \mu = \mu_j)\, p(\mu = \mu_j)}{\sum_{s=1}^{k} p(x = x_i \mid \mu = \mu_s)\, p(\mu = \mu_s)}$$

$$= \frac{\cancel{\frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu}_j)^T \Sigma^{-1} (\vec{x}-\vec{\mu}_j)} \quad p(\mu = \mu_j)}{\sum_{s=1}^{k} \cancel{\frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu}_s)^T \Sigma^{-1} (\vec{x}-\vec{\mu}_s)} \quad p(\mu = \mu_s)}$$

vs. $m_{ij}$ Hard
assignment in
K-means

# Learning a Gaussian Mixture

when assuming with known shared covariance

M-Step

$$\mu_j^{(t+1)} \leftarrow \frac{1}{\sum_{i=1}^{n} E[z_{ij}]} \sum_{i=1}^{n} E[z_{ij}]^{(t)} x_i$$

$$p(\mu = \mu_j)^{(t+1)} \leftarrow \frac{1}{n} \sum_{i=1}^{n} E[z_{ij}]^{(t)}$$

Covariance: $\sum_j$ (j: 1 to K) can also be derived in the M-step under a full setting

# M-step (vs. Centroid Step in K-means)

when assuming with known shared covariance

k mean => centroid $= \frac{1}{N_j} \sum_{i=1} m_{ij} X_i$

M-Step

$$\mu_j^{(t+1)} \leftarrow \frac{1}{\sum_{i=1}^{n} E[z_{ij}]} \sum_{i=1}^{n} \underbrace{E[z_{ij}]^{(t)}}_{\rightarrow [0,1]} x_i$$

$$\sum_{j=1}^{K} E[z_{ij}] = 1$$

$$p(\mu = \mu_j)^{(t+1)} \leftarrow \frac{1}{n} \sum_{i=1}^{n} E[z_{ij}]^{(t)}$$

Covariance: $\Sigma_j$ (j: 1 to K) will also be derived in the M-step under a full setting

# M-step for Estimating unknown Covariance Matrix (more general, details in EM-Extra lecture)



$$\Sigma_j^{(t+1)} = \frac{\sum_{i=1}^{n} E[z_{ij}]^{(t)} (x_i - \mu_j^{(t+1)})(x_i - \mu_j^{(t+1)})^T}{\sum_{i=1}^{n} E[z_{ij}]^{(t)}}$$

for small TrainSet
too many parameters
to estimate

$j = 1, \ldots, K$

$\Sigma_j \Rightarrow O(p^2/2)$

$\Sigma_j$ $\Rightarrow O(Kp^2/2)$

$\mu_j$ $O(Kp + K)$ $p(u=\mu_j)$

$E[z_{ij}]$ $O(Kn)$

# Recap: Expectation-Maximization for training GMM

- Start:
  - "Guess" the centroid and covariance for each of the K clusters
  - "Guess" the proportion of clusters, e.g., uniform prob 1/K
- Loop
  - For each point, revising its proportions belonging to each of the K clusters
  - For each cluster, revising both the mean (centroid position) and covariance (shape)

# Partitional : Gaussian Mixture Model

- 1. Review of Gaussian Distribution
- 2. GMM for clustering : basic algorithm
- 3. GMM connecting to K-means
- 4. Problems of GMM and K-means

# Recap: K-means iterative learning

$$\underset{\{\vec{C}_j, m_{i,j}\}}{\arg\min} \sum_{j=1}^{K} \sum_{i=1}^{n} m_{i,j} \left( \vec{x}_i - \vec{C}_j \right)^2$$

Memberships $\{m_{i,j}\}$ and centers $\{C_j\}$ are correlated.

E-Step Given centers $\{\vec{C}_j\}$, $m_{i,j} = \begin{cases} 1 & j = \arg\min_k (\vec{x}_i - \vec{C}_j)^2 \\ 0 & \text{otherwise} \end{cases}$

M-Step Given memberships $\{m_{i,j}\}$, $\vec{C}_j = \dfrac{\sum_{i=1}^{n} m_{i,j} \vec{x}_i}{\sum_{i=1}^{n} m_{i,j}}$

# Compare: K-means

- The EM algorithm for mixtures of Gaussians is like a "soft version" of the K-means algorithm.
- In the K-means "E-step" we do hard assignment:
- In the K-means "M-step" we update the means as the weighted sum of the data, but now the weights are 0 or 1:

K-means: $$\arg\min_{\{\vec{C}_j, m_{i,j}\}} \underbrace{\sum_{j=1}^{K}}\underbrace{\sum_{i=1}^{n}} m_{i,j}\left(\vec{x}_i - \vec{C}_j\right)^2$$

$m_{ij} = \begin{cases} 0 \\ 1 \end{cases}$

GMM : $$\sum_i \log \prod_{i=1}^{n} p(x = x_i) \overset{i=1..n}{=} \sum_i \log\left[ \overset{j=1,\cdots k}{\sum_{\mu_j}} p(\mu = \mu_j) \frac{1}{(2\pi)|\Sigma|^{1/2}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu}_j)^T \Sigma^{-1}(\vec{x}-\vec{\mu}_j)} \right]$$

- K-Mean only detect spherical clusters.
- GMM can adjust its self to elliptic shape clusters.

# (3) GMM Clustering



Data
↓
Task
↓
Representation
↓
Score Function
↓
Search/Optimization
↓
Models, Parameters

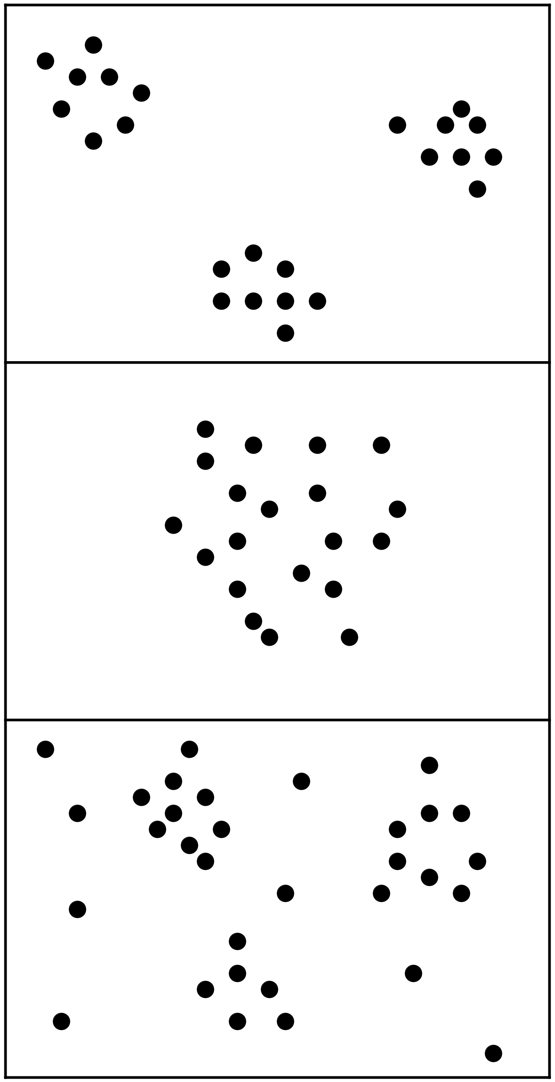
Tabular
↓
Clustering
↓
Mixture of Gaussian
↓
Likelihood
↓
EM algorithm
↓
Each point's soft membership & mean / covariance per cluster

$$\sum_i \log \prod_{i=1}^{n} p(x=x_i) = \sum_i \log \left[ \sum_{\mu_j} p(\mu=\mu_j) \frac{1}{(2\pi)\left|\Sigma_j\right|^{1/2}} e^{-\frac{1}{2}\left(\vec{x}-\vec{\mu}_j\right)^T \Sigma_j^{-1}\left(\vec{x}-\vec{\mu}_j\right)} \right]$$

# Partitional : Gaussian Mixture Model

- 1. Review of Gaussian Distribution
- 2. GMM for clustering : basic algorithm
- 3. GMM connecting to K-means
- 4. Problems of GMM and K-means

# Unsupervised Learning: not as hard as it looks



Sometimes easy

Sometimes impossible

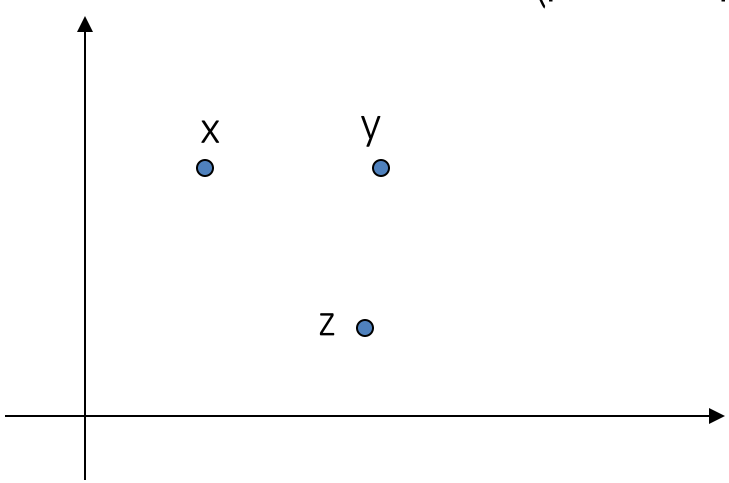
and sometimes in between

# Problems (I)

- Both k-means and mixture models need to compute centers of clusters and explicit distance measurement
  - Given strange distance measurement, the center of clusters can be hard to compute

E.g.,

$$\left\| \vec{x} - \vec{x}' \right\|_\infty = \max\left( \left| x_1 - x_1' \right|, \left| x_2 - x_2' \right|, \ldots, \left| x_p - x_p' \right| \right)$$




$$\left\| \mathbf{x} - \mathbf{y} \right\|_\infty = \left\| \mathbf{x} - \mathbf{z} \right\|_\infty$$

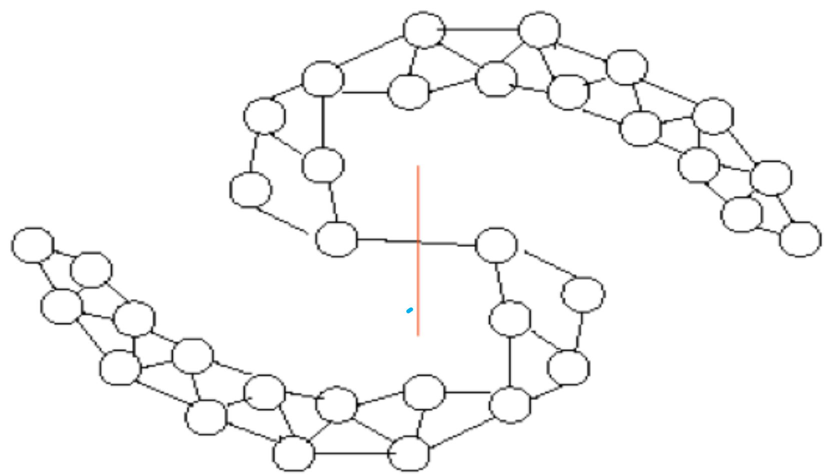
# Problem (II)

tight

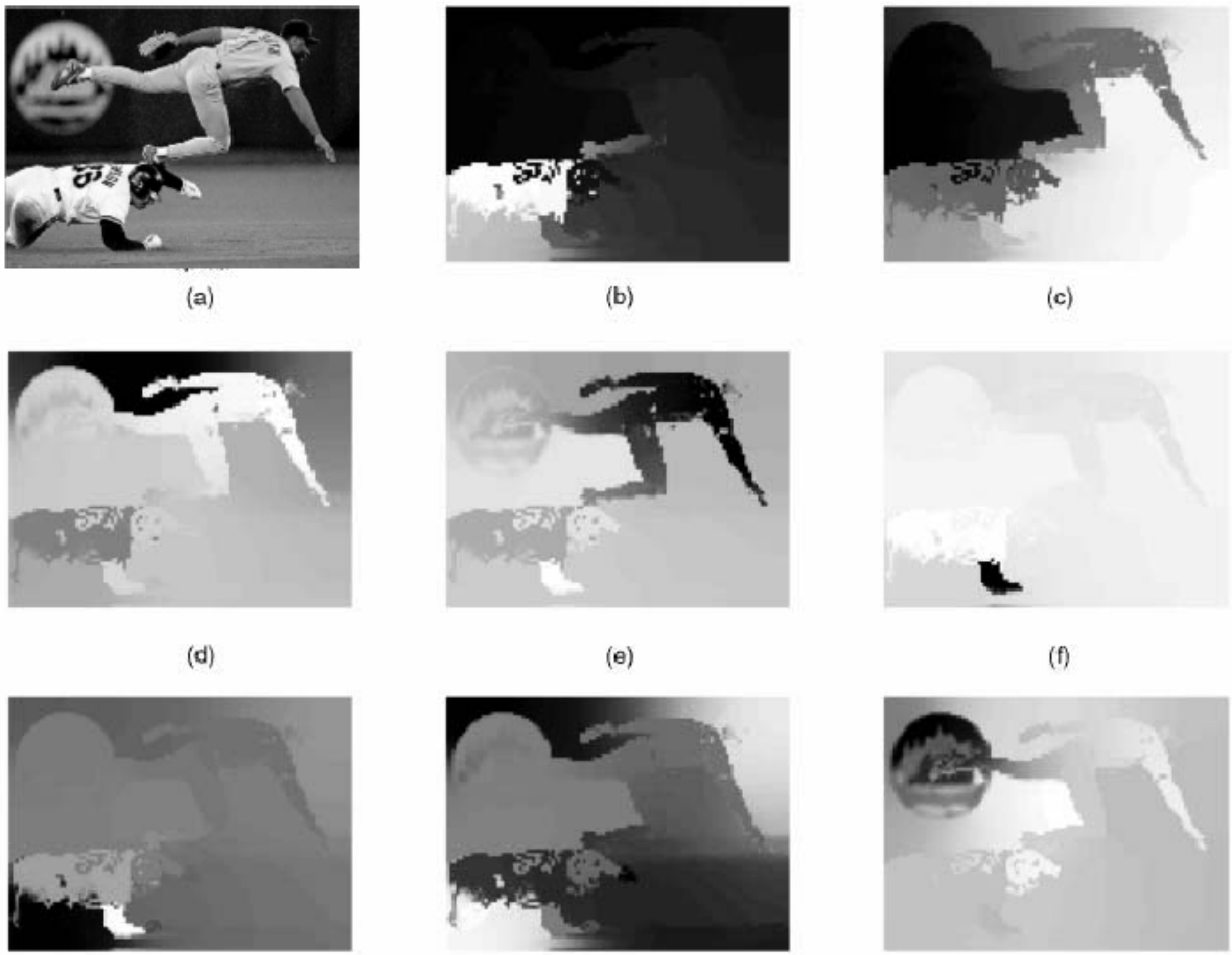
- Both k-means and mixture models look for compact clustering structures
  - In some cases, connected clustering structures are more desirable

Graph based clustering

e.g. MinCut, Spectral clustering



# e.g. Image Segmentation through minCut



# References

- Hastie, Trevor, et al. *The elements of statistical learning*. Vol. 2. No. 1. New York: Springer, 2009.
- Big thanks to Prof. Eric Xing @ CMU for allowing me to reuse some of his slides
- Big thanks to Prof. Ziv Bar-Joseph @ CMU for allowing me to reuse some of his slides
- clustering slides from Prof. Rong Jin @ MSU